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LETTER TO THE EDITOR

THERMAL FLAT-PLATE BOUNDARY-LAYER SOLUTIONS FOR A TWO-PHASE SUSPENSION WITH A FINITE VOLUME FRACTION

The problem of steady boundary-layer flow of a particulate suspension past a semi-infinite flat plate has been investigated by many authors (e.g. Prabha & Jain 1980; Osiptsov 1980; Chamkha & Peddieson 1991). All these investigators concluded that the particle-phase volume fraction becomes infinite at the plate surface. This conclusion violates the small volume fraction assumption inherent in the dusty-gas model employed in the analysis of this problem. Chamkha & Peddieson (1991) showed that this singularity can be eliminated by the inclusion of particle-phase diffusivity in the dusty-gas equations.

Recently, Chamkha & Peddieson (1992) employed a more general model allowing for finite volume fractions and particle-phase stresses and found, in contrast to the work mentioned above, that the volume fraction is constant in the boundary layer. This conclusion was obtained by using the most obvious order-of-magnitude analysis in developing the boundary-layer equations. Based on this model, solutions for the problem titled above are reported herein. The fluid phase is assumed to be incompressible and has constant properties. It is also assumed that there is no radiative heat transfer from one particle to another.

The dimensionless boundary-layer form of the energy equations can be written as

$$\begin{gathered}\partial_{\eta\eta}H - \mathrm{Pr}G\partial_\eta H + \mathrm{PrEc}(\partial_\eta F)^2 - 2\xi(1-\xi)\mathrm{Pr}F\partial_\xi H \\ + 2\xi\kappa\mathrm{Pr}(\gamma\epsilon(H_p - H) + \mathrm{Ec}(F_p - F)^2)/(1-\xi) = 0, \\ G_p\partial_\eta H_p - \mathrm{Ec}\beta/\gamma(\partial_\eta F_p)^2 + 2\xi(1-\xi)F_p\partial_\xi H_p - 2\xi\epsilon(H - H_p)/(1-\xi) = 0.\end{gathered} \qquad [1]$$

In [1], ξ and η are dimensionless tangential and normal coordinates, respectively; F and H are the fluid-phase tangential velocity and temperature (nondimensionalized by the free stream temperature), respectively; F_p, G_p and H_p are the particle-phase tangential velocity, transformed normal velocity and temperature, respectively. β, κ, Pr, Ec, γ and ϵ are the viscosity ratio, particle loading, fluid-phase Prandtl number, Eckert number, specific heat ratio and the temperature inverse Stokes number, respectively.

The boundary and matching conditions are

$$H(\xi, 0) = H_0, \quad H(\xi, \eta) \to 1, \quad H_p(\xi, \eta) \to 1 \text{ as } \eta \to \infty, \qquad [2]$$

where H_0 is a dimensionless wall temperature.

The wall heat transfer coefficient is defined as

$$\mathring{q}_w(\xi) = -\partial_\eta H(\xi, 0)/(\mathrm{PrEc}). \qquad [3]$$

Some numerical results of [1]–[3] [using the flow solutions given by Chamkha & Peddieson (1992)] are reported graphically in figures 1–3 to illustrate the influence of κ, β and n (a constant related to particulate wall slip) on $\mathring{q}_w$, respectively. It can be seen from figure 1 that the wall heat transfer increases as the particle loading increases. This is due to the increase in the interaction between the two phases in which the fluid gains kinetic and thermal energy from the particles. Figure 2 shows that increases in the values of β cause a decrease in the values of $\mathring{q}_w$ over a small range of the wall position (where particle-phase wall slip exists) followed by an increase in the values of $\mathring{q}_w$ over most of the plate (where a no-slip condition on the particle phase exists). It should be mentioned that the rate of transition from perfect particulate slip at $\xi = 0$ to a no-slip condition at $\xi = 1$ is controlled by the values of ω_0 [particulate wall slip coefficient, see Chamkha & Peddieson (1992)] and n. It can be seen in figure 3 that the dips in $\mathring{q}_w$ move upstream as n decreases. This is associated with the fact that the region of significant particulate wall slip decreases with decreasing n.

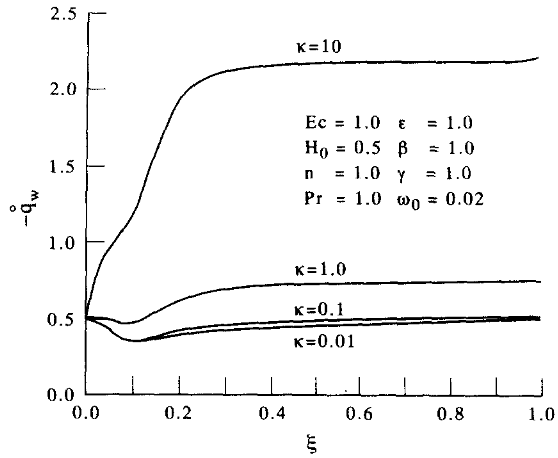

Figure 1. Wall heat transfer coefficient vs position.

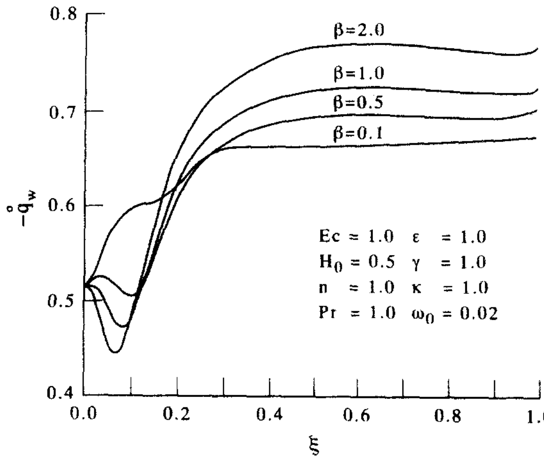

Figure 2. Wall heat transfer coefficient vs position.

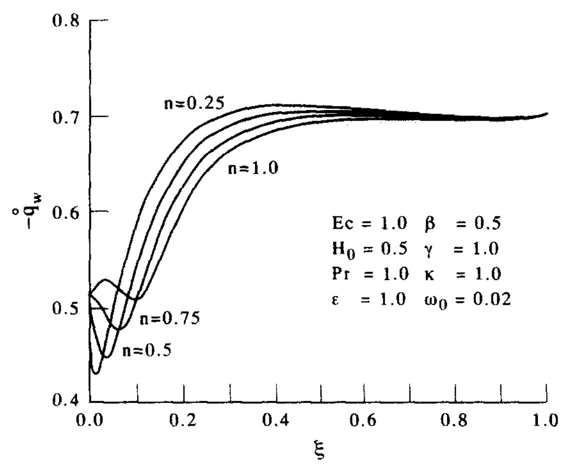

Figure 3. Wall heat transfer coefficient vs position.

The conclusion that the particle volume fraction is constant in the boundary layer represents one (of many) way of modeling this problem depending on the order-of-magnitude assumptions used to develop the governing equations. In the absence of experimental data, it is difficult to evaluate the validity of these assumptions. It is hoped that the flexibility that the present model offers will serve as a stimulus for experimental work and a useful vehicle for the investigation of alternate particle-phase stress models.

REFERENCES

Chamkha, A. J. & Peddieson, J. 1992 Flat plate boundary layer solutions for a particulate suspension with a finite volume fraction. *Dev. Theor. Appl. Mech.* **16,** II4.2–II4.7.

Chamkha, A. J. & Peddieson, J. 1991 Boundary layer flow of a particulate suspension past a flat plate. *Int. J. Multiphase Flow* **17,** 805–808.

Osiptsov, A. N. 1980 Structure of the laminar boundary layer of a disperse medium on a flat plate. *Fluid Dynam.* **15,** 512–517.

Prabha, S. & Jain, A. C. 1980 On the use of compatibility conditions in the solution of gas particulate boundary layer equations. *Appl. Scient. Res.* **36,** 81–91.

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